

STATISTICS ON BARGRAPHS OF CATALAN WORDS

DAVID CALLAN^(A) TOUFIK MANSOUR^(B) JOSÉ L. RAMÍREZ^(C,D)

^(A) *Department of Statistics, University of Wisconsin-Madison
1300 University Ave, Madison, WI 53706-1532, USA
callan@stat.wisc.edu*

^(B) *Department of Mathematics, University of Haifa
3498838 Haifa, Israel
tmansour@univ.haifa.ac.il*

^(C) *Departamento de Matemáticas, Universidad Nacional de Colombia
11001, Bogotá, Colombia
jlramirezr@unal.edu.co*

ABSTRACT

Catalan words are a particular case of growth-restricted words. Here we give the bivariate generating function for bargraphs associated to Catalan words according to the semiperimeter and area statistics. Exact formulas to count Catalan bargraphs according to the two statistics are also found. We show a connection to the Narayana numbers. Finally, we give similar results for the exterior corner statistic.

Keywords: Catalan word, generating function, area, semiperimeter, external corners

1. Introduction

A word $w = w_1w_2 \cdots w_n$ over the set of positive integers is called a *Catalan word* if $w_1 = 1$ and $1 \leq w_i \leq w_{i-1} + 1$ for $i = 2, \dots, n$. Let \mathcal{C}_n denote the set of the Catalan words of length n . For example, $\mathcal{C}_3 = \{111, 112, 121, 122, 123\}$. The cardinality of the set \mathcal{C}_n is given by the Catalan number $C_n = \frac{1}{n+1} \binom{2n}{n}$, see [25, Exercise 80]. There is a natural way to associate a Dyck path of length $2n$, or semilength n , with a Catalan word in \mathcal{C}_n . Recall that a *Dyck path* of semilength n is a lattice path of $\mathbb{Z} \times \mathbb{Z}$ running from $(0, 0)$ to $(2n, 0)$ that never passes below the x -axis and consists of n up diagonal steps $U = (1, 1)$ and n down diagonal steps $D = (1, -1)$, called *rise* and *fall*, respectively. An *ascent* in a Dyck path is a maximal sequence of consecutive rises and analogously for a *descent*. To a Dyck path of semilength n , we associate the word formed by the y -coordinate of each final point of the rise steps. This word is clearly a Catalan word of length n and the construction is a bijection. For example, in Figure 1, we show the Dyck path associated to the Catalan

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