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FORBIDDEN PATTERNS FOR ORDERED AUTOMATA

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ABSTRACT

The contribution concerns decision procedures in the algebraic theory of regular languages. Among others, various versions of forbidden patterns or configurations in automata are treated in the existing literature. Basically, one looks for certain subgraphs of the minimal automaton of a given language to decide whether this language does not belong to a given significant class of regular languages. We survey numerous known examples and we build a general theory covering the most of familiar ones. The chosen formalism differs from existing ones and the generalization to ordered automata enables us to reformulate some of known examples in a uniform shape. We also describe certain sufficient assumptions on the forbidden pattern which ensure that the corresponding class of languages forms a robust class in the sense of natural closure properties.

Keywords: deterministic automata, varieties of regular languages, varieties of automata

1. Introduction

Certain significant classes of regular languages can be characterized by some kind of forbidden patterns, which cannot occur in an automaton recognizing the language. To recall some examples, we can mention results by Cohen, Perrin and Pin [4] concerning the restriction of linear temporal logic obtained by considering only the operators "next" and "eventually". The useful characterization obtained in that paper is that a language L is expressible by this logic, denoted by RTL, if and only if the minimal automaton of L does not contain the pattern from Figure 1. This characterization gave a polynomial time algorithm for testing whether the language recognized by an n-state deterministic automaton is RTL-definable (see Theorem 4.2 and its Corollary 4.3 in [4]). The technique of forbidden patterns was also used by Schmitz et al. [5, 16, 17] for the first levels of the dot-depth hierarchy of the star-free languages.

This paper is focusing on formal theory of forbidden patters for deterministic finite automata, for which early formalisms were given in [4, 16]. For the purpose of this paper, the basic notion is a *semiautomaton* which is a deterministic automaton without initial and final states being specified. Then a pattern is an (incomplete)

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