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MISSING FACTORS OF IDEALS AND SYNCHRONIZING AUTOMATA

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ABSTRACT

Recently, a series of papers have started to look at Černý's conjecture, and in general at synchronizing automata, from the point of view of the theory of ideals of free monoids. The starting point of such an approach is a simple observation: the set of reset words of an automaton is a two-sided ideal of the free monoid on its alphabet that is also a regular language. We study the relationship between a synchronizing automaton and the sets of (minimal) generators of its reset words. We show that if such set does not contain a word of a certain length, then Černý's conjecture holds.

Keywords: synchronizing automaton, strongly connected automaton, Černý's conjecture, minimal reset word, ideals of free monoid

1. Introduction

A complete deterministic semiautomaton is a tuple $\mathcal{A} = (Q, \Sigma, \delta)$, where Q is a finite set of states, Σ is a finite alphabet acting on Q, and $\delta : Q \times \Sigma \to Q$ is a function that describes the action of Σ on the set Q. We use the notation $q.a = q'$ whenever $\delta(q, a) = q'$ and we extend this notation to words in Σ^* in the obvious way. Semiautomata are mostly used in theoretical computer science as language recognizers: by pinpointing an initial state q_0 and a set of final states $F \subseteq Q$, we get a deterministic finite automaton (DFA for short) $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ that defines the regular language $L(\mathcal{A}) = \{ u \in \Sigma^* \mid q_0.u \in F \}$ (see for instance [12]). Nevertheless, in this paper, we are interested in automata from a combinatorial point of view, and not as language recognizers, and for this reason in the rest of the paper we will refer to complete deterministic semiautomata simply with the name of automata. Our interest is mostly motivated by the longstanding Černý's conjecture regarding the class of synchronizing automata. An automaton is called synchronizing (or reset) whenever there is a word $u \in \Sigma^*$, called *reset word*, satisfying $q.u = q'.u$ for all $q, q' \in Q$. Černy's conjecture states that a synchronizing automaton with n states has always a reset word of length at most $(n-1)^2$ (see [5]). The literature around Černý's conjecture and synchronizing automata is quite impressive, and spans from the algorithmic point