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EXTENSIONS TO MINIMAL SYNCHRONIZING WORDS

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ABSTRACT

Extension problems have been studied for a variety of combinatorial properties, but not so for combinatorial questions on formal languages. We will raise this type of question for one of the undoubtedly most prominent properties of words in relation to finite automata, namely their synchronizability. In contrast to other areas of discrete mathematics, say, to graph theory, there are several natural ways how to define extension problems for synchronizing words, depending on the chosen partial order on the set of all words. Some variants lead to polynomial-time solvable extension problems, while others yield (co-)NP-hard extension problems, and still others lead to open problems. We also take a closer look at commutative automata and show that the combinatorics of synchronizing words is much easier for this class, while the related computability problems are still NP-hard.

Keywords: synchronizing word, extension problem, commutative automata

1. Introduction

We assume that the reader is familiar with the basics of deterministic finite automata, or DFAs for short. Recall that a DFA A can be specified by its state alphabet S, its input alphabet Σ, the total[1] transition function $\delta : S \times \Sigma \to S$, a designated start state s_0 and a set of final states F. In fact, for our purposes, the specification of s_0 and F is not important. As usual, the transition function can be inductively extended to $\delta^* : S \times \Sigma^* \to S$. For states $s_1, s_2 \in S$, let

$$L_{s_1,s_2}(A) = \{ \; w \in \Sigma^* \mid \delta^*(s_1, w) = s_2 \; \}.$$

A word $x \in \Sigma^*$ is called *synchronizing* for a DFA A, $A = (S, \Sigma, \delta, s_0, F)$, if there is a state s_f, also called the *synchronizing state* of A, such that for all states $s \in S$,

$$\delta^*(s, x) = s_f.$$

Let $L_{\text{sync}}(A)$ collect all synchronizing words for A. The DFA A is called *synchronizable* if $L_{\text{sync}}(A) \neq \emptyset$. In the context of this Special Issue, there is no need to explain the

[1] If nothing else is said, the DFAs considered in this paper will be assumed to be complete.