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ON THE DENSITY OF LANGUAGES ACCEPTED BY TURING MACHINES AND OTHER MACHINE MODELS

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ABSTRACT

A language is dense if the set of all infixes (or subwords) of the language is the set of all words. Here, it is shown that it is decidable whether the language accepted by a nondeterministic Turing machine with a one-way read-only input and a reversal-bounded read/write worktape (the read/write head changes direction at most some fixed number of times) is dense. From this, it is implied that it is also decidable for one-way reversal-bounded queue automata, one-way reversal-bounded stack automata, and one-way reversal-bounded k-flip pushdown automata (machines that can "flip" their pushdowns up to k times). However, it is undecidable for deterministic Turing machines with two 1-reversal-bounded worktapes (even when the two tapes are restricted to operate as 1-reversal-bounded pushdown stacks).

Keywords: density, Turing machines, store languages, pushdowns, queues, stacks

1. Introduction

A language $L \subseteq \Sigma^*$ is said to be *dense* if the set of all infixes of L is equal to Σ^* [3]. This is an interesting property especially relevant to the theory of codes [15]. The notion has been investigated as it pertains to independent sets, maximal independent sets, and disjunctive languages [13, 18]. Later, the notion was generalized from the set of infixes of a language being the universe, to arbitrary relations used in place of the infix relation [14]. For example, a language L is suffix-dense if the set of all suffixes of L is equal to the universe. Homomorphisms that preserve different types of density were investigated as well [16].

Recently, these generalized notions of density were studied as applied to types of pushdown automata and counter machines [4]. It was surprisingly found that it is

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