

ON THE STRUCTURE OF FREE ITERATION SEMIRINGS

ZOLTÁN ÉSIK¹

*Dept. of Computer Science
University of Szeged, Hungary
e-mail: ze@inf.u-szeged.hu*

and

TAMÁS HAJGATÓ²

*Dept. of Computer Science
University of Szeged, Hungary
e-mail: hajgato@inf.u-szeged.hu*

ABSTRACT

We give a characterization of the free iteration semirings by rational series.

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1. Introduction

Iteration semirings were defined in [2] as semirings $S = (S, +, \cdot, 0, 1)$ equipped with a star operation $*$: $S \rightarrow S$ subject to certain equational axioms such as the fixed point equation $x^* = xx^* + 1$. Thus, iteration semirings form a variety as defined in universal algebra. A simplified system of axioms appears in [8, 9]. Fundamental examples of iteration semirings are the continuous and complete semirings [7, 2], the semirings of (regular) languages, the semirings of (rational) power series with coefficients in the semiring \mathbb{N}_∞ (i.e., the semiring \mathbb{N} of nonnegative integers endowed with a point ∞ at infinity). It has been shown that an equation between rational (or regular) terms holds in all semirings of (regular) languages iff it holds in all iteration semirings satisfying the equation $1^* = 1$, cf. [11, 2]. Moreover, as shown in [4], an equation holds in all complete semirings iff it holds in all continuous semirings iff it holds in all semirings of (rational) power series with coefficients in \mathbb{N}_∞ iff it holds in all iteration semirings satisfying three additional equations: $1^*x = x1^*$, $(1^*x)^+ = 1^*x^+$ and $1^{**} = 1^*$.

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