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MINICOMPLEXITY [1]

Christos A. Kapoutsis [2]
LIAFA, Université Paris VII, France
e-mail: christos.kapoutsis@liafa.univ-paris-diderot.fr

ABSTRACT

This is an overview of *minicomplexity*, i.e., of the *complexity of two-way finite automata*. We start with a smooth introduction to its basic concepts, which also brings together several seemingly detached old theorems. We then record some recent advances, both in the theory itself and in its relation to the space complexity of Turing machines. Our exposition follows, extends, and advocates the classic framework of Sakoda and Sipser.

Keywords: Two-way finite automata, descriptional complexity, state complexity, minicomplexity, Turing machines, space complexity

1. Introduction

In Theory of Computation, the distinction between *computability* and *complexity* is clear. In computability, we ask whether a given problem can be solved by a Turing machine (TM), namely whether the problem is *computable*. In complexity, we focus exclusively on problems that indeed can be solved, and we ask how much of a TM's resources they require, the main resources of interest being *time* and *space*.

This distinction is also valid for finite automata (FAs). In FA-*computability*, we ask whether a problem can be solved by a FA; often, but not always, this is the same as asking whether the problem is *regular*. In FA-*complexity*, we focus exclusively on problems that indeed can be solved, and we ask how much of a FA's resources they require; often, but not always, the resource of interest is *size* (as expressed, e.g., by the number of states). Hence, much like the theory of TMs, the theory of FAs also consists of a computability and a complexity component.

This distinction is not widely realized. Specifically, the complexity component is often overlooked. Standard textbooks essentially identify the entire theory of FAs with FA-computability (e.g., see [30, Chapter 1]), barely addressing any FA-complexity issues (e.g., as in [30, Problems 1.60-1, 1.65]). Perhaps one might try to justify this systematic neglect by claiming that these issues are not really a theory; they are just a

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