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SYNTACTIC COMPLEXITIES OF SIX CLASSES OF STAR-FREE LANGUAGES [1]

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ABSTRACT

The syntactic complexity of a regular language is the cardinality of its syntactic semigroup. The syntactic complexity of a subclass of regular languages is the maximal syntactic complexity of languages in that subclass, taken as a function of the state complexity n of these languages. We study the syntactic complexity of six subclasses of star-free languages. We find a tight upper bound of $(n-1)!$ for finite/cofinite and reverse definite languages, and a lower bound of $\lfloor \mathbf{e} \cdot (n-1)! \rfloor$ for definite languages, where $\mathbf{e}$ is the base of the natural logarithms. We also find tight upper bounds for languages accepted by monotonic, partially monotonic and "nearly monotonic" automata. All these bounds are significantly lower than the bound n^n for arbitrary regular languages. Also, witness languages reaching these bounds require alphabets that grow with n. The syntactic complexity of arbitrary star-free languages remains open.

Keywords: cofinite language, definite language, finite automaton, finite language, monotonic automaton, partially monotonic automaton, reverse definite language, star-free language, syntactic complexity, syntactic semigroup

1. Introduction

Star-free languages are the smallest class containing the finite languages and closed under boolean operations and concatenation. In 1965, Schützenberger [26] proved that a language is star-free if and only if its syntactic monoid is *group-free*, that is, has only trivial subgroups. An equivalent condition is that in the minimal deterministic

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