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COUNTING PRIMITIVE PARTIAL WORDS [1]

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ABSTRACT

A word is primitive if it is not a power of another word. The number of primitive words of a fixed length over an alphabet of a fixed size is well known and relates to the Möbius function. In this paper, we investigate the number of primitive partial words which are strings that may contain "do not know" symbols.

Keywords: Combinatorics on words, words, partial words, primitive words, primitive partial words, Möbius function, periods, exact periods

1. Introduction

Primitive words, or strings over a finite alphabet that cannot be written as a power of another string, play an important role in numerous research areas including formal language theory [16, 17], coding theory [4, 26], and combinatorics on words [14, 20, 21, 22, 23]. *Partial words* (or *pwords*) are strings that may contain a number of "do not know" symbols also called "holes" (words, or *full words*, are partial words without holes). *Primitive partial words* were defined in [5]. A partial word u is primitive if there exists no word v such that $u \subset v^i$ with $i \geq 2$ (the concept of containment, denoted by $\subset$, is discussed in Section 2). Testing whether or not a partial word is primitive can be done in linear time in the length of the word [6]. This result, which extends a result on words [15], found a nice application in [9]. There, Blanchet-Sadri and Chriscoe extend to partial words with one hole the well known result of Guibas and Odlyzko [19] which states that the sets of periods of words are

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