

A NOTE ON DEFECT THEOREMS FOR 2-DIMENSIONAL WORDS AND TREES¹

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ABSTRACT

Defect theorem, a classical result of combinatorics on words, can be extended on 2-dimensional words and trees under some restrictions. In this note we answer to two open problems stated in W. Moczurad, (Theor. Inform. Appl. 41, 2007): whether the defect property holds or does not hold for three rectangles and for two figures of unrestricted shape. We give two counterexamples to show that the defect property fails in both situations. This completes the analysis of the defect property with 2-dimensional words in the classes of dominoes, squares, rectangles and figures with unrestricted shape. In addition, we examine the same question of the validity of the defect property with trees.

Keywords: Defect theorem, 2-dimensional words, trees

1. Introduction

The classical version of the defect theorem is usually stated in the context of words: if a set of $n + 1$ words satisfies a nontrivial equation, that is, the set is a non-code, then these $n + 1$ words can be expressed with at most n words by concatenating these (*cf.* Lothaire [3, 4]). This one of the fundamental results of combinatorics on words can also be extended to other contexts. The notion of defect property has been examined, for example, among the sets of trees and figures [1, 2, 6, 7]. Although the property holds for trees, it is not satisfied by figures in general. For example, in [1, 7] there were given some boundaries to the size of a set where the property fails among figures of certain shape. In chapter 2 we fix some notation and state earlier results and in chapter 3 we complete this analysis of the defect theorem of figures by giving two counterexamples of sets which do not satisfy the property. Thus, we can establish accurate boundaries for the validity of the defect property among dominoes, squares, rectangles and figures with unrestricted shape. In the last chapter we examine the corresponding boundary size of a tree set for which the defect property is not valid in connection with prefix rank.

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