

CHURCH-ROSSER GROUPS AND GROWING CONTEXT-SENSITIVE GROUPS

MARK KAMBITES¹

*School of Mathematics, University of Manchester
Manchester M13 9PL, England
e-mail: mark.kambites@manchester.ac.uk*

and

FRIEDRICH OTTO²

*Fachbereich Elektrotechnik/Informatik, Universität Kassel
D-34109 Kassel, Germany
e-mail: otto@theory.informatik.uni-kassel.de*

ABSTRACT

A finitely generated group is called a *Church-Rosser group* (*growing context-sensitive group*) if it admits a finitely generated presentation for which the word problem is a Church-Rosser (growing context-sensitive) language. Although the Church-Rosser languages are incomparable to the context-free languages under set inclusion, they strictly contain the class of deterministic context-free languages. As each context-free group language is actually deterministic context-free, it follows that all context-free groups are Church-Rosser groups. As the free abelian group of rank 2 is a non-context-free Church-Rosser group, this inclusion is proper. On the other hand, we show that there are co-context-free groups that are not growing context-sensitive. Also some closure and non-closure properties are established for the classes of Church-Rosser and growing context-sensitive groups. More generally, we also establish some new characterizations and closure properties for the classes of Church-Rosser and growing context-sensitive languages.

Keywords: Church-Rosser language, growing context-sensitive language, word problem, co-word problem

1. Introduction

Let $\langle \Sigma; R \rangle$ be a finitely generated presentation of a group G , that is, Σ is a finite set of generators and $R \subset \underline{\Sigma}^*$ is a set of defining relators, where $\overline{\Sigma}$ is a set of formal inverses that is in one-to-one correspondence to Σ , and $\underline{\Sigma} = \Sigma \cup \overline{\Sigma}$. With this presentation we can associate the language $\text{WP}(G, \Sigma)$, which consists of all words over $\underline{\Sigma}$ that

¹The work of this author was conducted at Universität Kassel, while he was supported by a Marie Curie Intra-European Fellowship within the 6th European Community Framework Programme.

²Corresponding author.