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A COMBINATORIAL STUDY OF k-VALUED RATIONAL RELATIONS

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ABSTRACT

We give combinatorial proofs for three fundamental properties of k-valued rational relations: the decomposition of a k-valued rational relation into a union of k rational functions; the decidability of the k-valuedness; the decidability of the equivalence for k-valued rational relations. Positive answers are already known for these properties. Our contribution lies in the fact that our proofs are built on few elementary combinatorial arguments, which make them considerably shorter and hopefully easier to understand. In particular, in order to tackle the k-valuedness, we generalize a combinatorial characterization of the rational functions due to Schützenberger.

Keywords: Finite-valued rational relation, k-valued rational relation, transducer, rational function, decidability

1. Introduction

Rational relations (between free monoids) and transducers are classical concepts in automata theory, and are considered in several references on the subject [8, 13, 2]. Among them, the family of the rational functions (and functional transducers) has received much attention due to the remarkable properties it bears, such as unambiguity and decidability. Indeed, whereas even some rather simple rational relations can be shown to be intrinsically ambiguous, every rational function can be realized by an unambiguous transducer, as shown by Eilenberg [8] and independently by Schützenberger [18]. (Another proof can be derived from the fundamental work of Elgot and Mezei [9].) And whereas the equivalence is undecidable for transducers, the

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