

ON THE CLOSURE OF RECOGNIZABLE TREE SERIES UNDER TREE HOMOMORPHISMS ¹

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ABSTRACT

We give an equational characterization of the closure of the class of recognizable tree series under non-deleting tree homomorphisms. Several applications are presented.

Keywords: Tree series, recognizability, substitution, homomorphisms

1. Introduction

According to Mezei and Wright [11] an equational subset of an arbitrary Γ -algebra \mathcal{A} is a component of the least solution of a system of regular equations. Mezei and Wright proved that the equational subsets of the Γ -algebra T_Γ of trees are exactly the recognizable tree languages and any equational subset of an arbitrary Γ -algebra is the homomorphic image of a recognizable tree language.

Engelfriet and Schmidt in [3, 4] use two basic modes of substitutions, the *IO* and *OI*, in order to solve context-free systems of equations. In the first one, the same object is substituted at all the occurrences of the same variable whereas in the second mode different objects may be substituted for different occurrences of the same variable. They prove that the class of *u*-equational solutions of context-free systems coincide with the class of *u*-context-free tree languages ($u = IO, OI$). Furthermore, they establish a series of ‘Mezei and Wright like’ theorems relating context-free and recognizable tree languages.

In this paper we deal with systems of equations whose right-hand side members are formal power tree series. Such a system admits two different kinds of solutions according to the substitution operation we use to solve it (*IO* and *OI*). Tree series obtained as components of least *u*-solutions of systems whose right-hand side members are polynomials are called *u*-equational ($u = IO, OI$). A classical result due to Berstel and Reutenauer [1] (see also [2, 12]) confirms that the class *OI*-Equat of *OI*-equational tree series coincides with the class *REC* of recognizable tree series. It is also well-known [2, 10] that linear non-deleting homomorphisms preserve recognizable tree

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