Journal of Automata, Languages and Combinatorics 9 (2004) 2/3, 365–381

STATE COMPLEXITY OF LINEAR CONJUNCTIVE LANGUAGES [1]

ALEXANDER OKHOTIN
School of Computing, Queen's University
Kingston, Ontario, Canada K7L3N6
e-mail: okhotin@cs.queensu.ca

ABSTRACT

The ε-free languages generated by linear conjunctive grammars have recently been proved to be exactly the languages accepted by trellis automata. This paper begins the study of the descriptional complexity of this language family by comparing the number of states in automata with the size of grammars. The state complexity of the languages $(a^C)^+$ and $\{a^n(b^{C^n})^+ \mid n \geq 1\}$ is determined (it is C and $C+3$ respectively), leading to an exact expression for the worst-case complexity of all set-theoretic operations and to the non-uniqueness of minimal automata. A superpolynomial lower bound and an exponential upper bound for the succinctness tradeoff between linear conjunctive grammars and trellis automata are established.

Keywords: Linear conjunctive grammars, trellis automata, descriptional complexity

1. Introduction

Conjunctive grammars [5] are a generalization of context-free grammars that allows the use of an explicit intersection operation, so that each production is of the form $A \to \alpha_1 \& \ldots \& \alpha_n$ and the conjunction ('&') has semantics of intersection of languages generated by each string α_i.

Linear conjunctive grammars [5, 6] constitute a special case of conjunctive grammars, in which all rules are of the form $A \to u_1 B_1 v_1 \& \ldots \& u_n B_n v_n$ $(n \geq 1)$ or $A \to w$, where A, B_i are nonterminal symbols and u_i, v_i, w are terminal strings. Every linear conjunctive language is in $\mathbf{DTIME}(n^2)$ [5], and the family of linear conjunctive languages is known to be closed under all set-theoretic operations [6]. Additionally, many important languages, such as $\{a^n b^n c^n \mid n \geq 0\}$ or $\{wcw \mid w \in \{a,b\}^*\}$, are known to be linear conjunctive.

The recently established computational equivalence [7] of linear conjunctive grammars to trellis automata [2] and one-way real-time cellular automata [1], together with the known equivalence of these devices to a certain type of sequential machines and

[1] Full version of a submission presented at the 4th Workshop on *Descriptional Complexity of Automata, Grammars and Related Structures* (London, Ontario, Canada, August 21–24, 2002).