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A CHARACTERIZATION OF COMPLETE FINITE PREFIX CODES IN AN ARBITRARY SUBMONOID OF A^*

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ABSTRACT

Given an arbitrary submonoid M of the free monoid A^*, and given a subset X of M, X is weakly M-complete if any word of M is a factor of some word in X^*. The submonoid X^* itself is weakly M-dense. We apply two results from [8, 9] for obtaining a new characterization of the existence of a finite weakly M-complete prefix set: such a set exists iff M itself is weakly dense in its right unitary hull. This leads to an efficient algorithmic for deciding whether a given finite prefix subset of a finitely generated submonoid M is (weakly) M-complete.

Keywords: Free monoid, submonoid, unitary, codes, prefix, suffix, bifix, complete, dense, maximal

1. Introduction

Completeness of sets plays a prominent part in the free monoid theory: this is due to its mathematical relevance just as much as its potential applications. A subset X of the free monoid A^* is complete if any word of A^* is a factor of some word of X^*, the submonoid generated by X. From a topological point of view, X is complete iff X^* (i. e. the submonoid generated by X) is dense in A^*, with respect to the topology which is generated by the family of bilateral ideals of A^* and the empty set. From an algebraical point of view, a famous result due to Schützenberger states that, for the remarkable families of thin codes or thin prefix codes [1, p. 65], completeness and maximality (with respect of the inclusion ordering) are two equivalent notions. It is of interest to note that, based on this property, many studies have been drawn for investigating whether the preceding equivalence holds in other special families of codes (see e. g. [13, 6, 3, 10, 7]).

In this paper we are interested in the family of sets that are included in an arbitrary submonoid $M \subseteq A^*$. We consider the following notion of weak completeness: a set $X \subseteq M$ is weakly M-complete if any word in M is a factor of some word in X^*. The notion of weakly M-completeness is in fact more general than the notion of M-completeness as introduced in [1], in which any word in M must be completed in X^* by making use of words of M: in the most general case, weak completeness allows