

ON DEFINITE AND NILPOTENT DR TREE LANGUAGES¹

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ABSTRACT

The sets recognized by deterministic root-to-frontier tree recognizers, the *DR tree languages*, are determined by their path languages. A path language of a tree language T consists of the words describing the paths leading from the root of a tree in T to its leaves labelled with a given leaf symbol. The greatest congruence saturating the path languages yields the *syntactic path monoid* of T which is finite for a path closed T exactly in case T is a DR tree language. We introduce the concepts of definite and nilpotent DR tree languages and, by means of syntactic path semigroups, characterize them.

Keywords: Automata, tree automata, languages, tree languages

1. Introduction

The family recognized by deterministic root-to-frontier (top-down) tree recognizers, the *DR tree languages*, forms a proper subfamily of the family of all regular tree languages. Any DR tree language T is *closed* in the sense that it contains every tree t such that any labelled path, in which also the direction taken at each node is indicated, leading from the root of t to a leaf with a given label, appears in some tree belonging to T . For each leaf symbol x from the leaf alphabet X , the *path language* T_x is defined as the set of all words over $\hat{\Sigma}$ describing a path in a tree in T from the root to an x -labelled leaf. In [5] the *syntactic path congruence* of a tree language T as the intersection of the syntactic congruences of the path languages of T , and the *syntactic path monoid* of T as the corresponding quotient monoid have been introduced. In the same paper it has been proved that a closed tree language is DR-recognizable iff its syntactic path monoid is finite. Furthermore, in [2] we have described monotone DR tree languages in terms of syntactic path monoids. In this study we introduce the concepts of definite and nilpotent DR tree languages and characterize them by means of syntactic path semigroups.

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