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HIGHER DIMENSIONAL AUTOMATA [1]

ZOLTÁN ÉSIK and ZOLTÁN L. NÉMETH
Department of Computer Science, University of Szeged
P. O. Box 652, H-6701 Szeged, Hungary
e-mail: {ze, zlnemeth}@inf.u-szeged.hu

ABSTRACT

We provide the basics of a 2-dimensional theory of automata on series-parallel biposets. We define recognizable, regular and rational sets of series-parallel biposets and study their relationship. Moreover, we relate these classes to languages of series-parallel biposets definable in monadic second-order logic.

Keywords: Automata on biposets, bisemigroups, recognizability, rationality, logical definability

1. Introduction

Finite automata process words, i. e., elements of a finitely generated free semigroup. In this paper, we define automata whose input structure is a finitely generated free bisemigroup equipped with two associative operations. The elements of the free bisemigroup may be represented by labelled series-parallel biposets. We introduce recognizable, regular and rational sets of series-parallel biposets and study their relationship. Moreover, by relying on the main result of Hoogeboom and ten Pas [17], we relate these classes to languages of series-parallel biposets definable in monadic second-order logic. All of our results can be generalized to higher dimensions, i. e., to any finite number of associative operations.

Our study owes much to the work of Hoogeboom and ten Pas [16, 17] on text languages, and to the recent work of Lodaya and Weil [20, 21] and Kuske [18, 19] on languages of series-parallel posets that may be seen as a two-dimensional extension of the classical theory to a situation where one of the two associative operations is commutative. We believe that the case that none of the two operations is commutative is more fundamental in the same way as automata on free monoids are more fundamental than automata on free commutative monoids. An independent study of automata and languages over free bisemigroups was also initiated by Hashiguchi

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