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MORE ON GENERALIZED AUTOMATIC SEQUENCES

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ABSTRACT

We give some generalizations of k-automatic sequences replacing the k-ary system by an abstract numeration system on a regular language. We study some of the closure properties of these sequences and the possible extension to the multidimensional case or to infinite alphabets. The equivalence of these sequences and morphic predicates is given and the relationship to recognizability is also investigated.

Keywords: Automatic sequences, regular sequences, numeration systems, recognizable sets, morphic predicates

1. Introduction

In the late sixties, A. Cobham introduced, in relation to the classical numeration systems with an integer base $k \geq 2$, the k-automatic sequences or uniform tag sequences. A *k-automatic sequence* can be defined by two kinds of processes. On the one hand, the nth term of the sequence is the result given by a deterministic finite automaton with output fed with the k-ary representation of n. On the other hand, the sequence is the image under a letter-to-letter morphism of the infinite word that is the fixed point of an iterated morphism of length k [7].

Even if these k-automatic sequences are of low complexity (to be precise the number of factors of length n in such a sequence is $\mathcal{O}(n)$) they are not necessarily ultimately periodic and they find applications in physics and in number theory (see for instance [1, 2]). In particular, these sequences are extensively used to study the k-recognizable sets of integers. (A set of integers is said to be *recognizable*, with respect to a given

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