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WORDS AVOIDING ABELIAN INCLUSIONS

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ABSTRACT

We study a generalization of abelian squares which we call *abelian inclusions*: a word uv is said to be an $f(l)$-inclusion if the commutative image of v majorizes that of u, and $|v| \leq |u| + f(|u|)$. We prove that cl-inclusions are unavoidable, but c-inclusions are avoidable for an arbitrary constant c.

Keywords: Abelian powers, commutative image, pattern avoidance

1. Introduction

In 1961, P. ERDÖS [4] posed a problem whether there exist infinite words on finite alphabets avoiding abelian squares, i. e., never containing successive occurrences of words equal up to a permutation of symbols. This problem arised from the theory of avoiding successive occurrences of equal words started in the works of A. THUE [12] and evolved to the theory of pattern avoidance [2].

The first answer to the ERDÖS's problem was given in 1968 by A. A. EVDOKIMOV [5] who found a word avoiding abelian squares on the 25-letter alphabet. The question of minimal possible cardinality of the alphabet arised naturally, and the advances in it were made by A. A. EVDOKIMOV [6] who reduced the number of symbols to 7 and by P. A. B. PLEASANTS [10] who lowered it to 5. However, for about 20 years it had not been known if there exists an abelian square-free word on the 4-letter alphabet; at last, in 1992 such a word was found by V. KERÄNEN [7]. Since it is relatively easy to check that each infinite word on the 3-letter alphabet contains an abelian square, KERÄNEN's result completed the solution of ERDÖS's problem.

Different generalizations and extensions of KERÄNEN's result have appeared. A related problem of avoiding words having the same frequency of each letter was considered in [8, 11]. A. CARPI [1] obtained the first estimate of the number $h(n)$ of words of length n on 4 letters avoiding abelian squares: he proved that

$$\lim_{n\to\infty} \inf h(n)^{1/n} > 1.000021.$$

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