

# SIZES OF CONTEXT-FREE LANGUAGES GENERATED BY CONTEXT-FREE GRAMMARS AND STABLE AND RECURRENT 0L SYSTEMS <sup>1</sup>

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## ABSTRACT

In this paper we consider the sizes of context-free languages with respect to context-free grammars and to some special 0L systems, i. e., 0L systems with adult, stable, and recurrent terminations. A word is  $k$ -stable if it only produces itself in at most  $k$  steps. A word is stable if it is  $k$ -stable for some  $k$  and a 1-stable word is called an adult word. A word is recurrent if it is produced by any of its descendants. Adult, stable, and recurrent languages for a 0L system are the sets of all adult, stable, and recurrent words derived by the system, respectively. We discuss the number of rules and the total length of rules as the sizes or the descriptive complexities of grammars and 0L systems. It is shown that for every context-free language the sizes of a 0L system generating the language as a recurrent language are smaller than the sizes of context-free grammars generating it. We show that there exists a subfamily of context-free languages in which the total length of rules of a 0L system generating each language of the subfamily as a recurrent language is at most the positive square root of that of any context-free grammar generating it.

*Keywords:* descriptive complexity, context-free grammar, stable 0L system, recurrent 0L system.

## 1. Introduction

Recurrent languages [7, 10] for 0L systems are generalizations of adult [3] and stable [1] languages for 0L systems. A word is said to be adult if it only produces itself by a given 0L system. A word  $u$  is  $k$ -stable if  $u$  appears in any derivation starting off with  $u$  in at most  $k$ -steps. A word is called stable if it is  $k$ -stable for some  $k$ . An adult (resp. stable) language for a 0L system consists of all adult (resp. stable) words generated from the axiom by the 0L system. A recurrent word  $u$  can be produced by any of its descendants, that is, for every word  $v$  produced by  $u$ ,  $v$  produces  $u$  in a given 0L system. A recurrent language for a 0L system consists of all recurrent words generated from the axiom by the 0L system. The families of adult, stable,

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