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INJECTIVITY OF THE QUOTIENT $h\backslash g$ OF TWO MORPHISMS AND AMBIGUITY OF LINEAR GRAMMARS

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ABSTRACT

From a linear context-free grammar simulating an arbitrary instance (φ_1, φ_2) of the Post correspondence problem PCP we easily construct two nonerasing morphisms h and g with h length-duplicating such that the instance (h, g) of PCP has no solution but (φ_1, φ_2) possesses a solution if and only if the quotient operation $h\backslash g$ is not injective on its domain. Hence, the undecidability of the injectivity problem follows.

Keywords: undecidability, morphism, injectivity, linear context-free grammars, Post correspondence problem.

1. Introduction

If a word u is a prefix of another word w, i.e., if $w = uv$ for some v, then the *overflow* $u^{-1}w$ is defined by $u^{-1}w = v$. Otherwise, the operation is undefined. For any morphisms $h, g \colon X^* \to Y^*$, we may define, for instance, the following operations

- pairing: $\langle h, g\rangle(x) = (h(x), g(x))$,
- duplication: $(h \circ g)(x) = h(x)g(x)$,
- quotient: $(h\backslash g)(x) = h(x)^{-1}g(x)$.

It is well-known that the injectivity of morphisms on regular languages is decidable. However it is undecidable whether an arbitrary pairing is injective on its domain (see [3] or [2]) and whether a duplication is injective on every set of all words of equal length. The latter assertion follows from [7, Theorem 4.1].

The quotient operation is used in [1] where it is shown that every recursively enumerable language $L \subseteq \Sigma^*$ can be presented in the form $L = (h\backslash g)(\Delta^+) \cap \Sigma^*$. A short proof of this result in [6] (or see [2]) also reproves that PCP and its modified version are undecidable. All languages of the form $(h\backslash g)(\Delta^+)$ are not recursive, but their emptiness problem is trivially decidable.

We shall show that the injectivity of quotients $h\backslash g$ is undecidable. This also gives another solution to the Erasing Writing PCP introduced in [8]. The solution given in [4] is based on a long construction of Ruohonen [5].