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QUASIPERIODS, SUBWORD COMPLEXITY AND THE SMALLEST PISOT NUMBER

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ABSTRACT

A quasiperiod of a finite or infinite string/word is a word whose occurrences cover every part of the string. A word or an infinite string is referred to as quasiperiodic if it has a quasiperiod. It is obvious that a quasiperiodic infinite string cannot have every word as a subword (factor). Therefore, the question arises how large the set of subwords of a quasiperiodic infinite string can be [8].

Here we show that on the one hand the maximal subword complexity of quasiperiodic infinite strings and on the other hand the size of the sets of maximally complex quasiperiodic infinite strings both are intimately related to the smallest Pisot number t_P (also known as *plastic constant*).

We provide an exact estimate on the maximal subword complexity for quasiperiodic infinite words.

Keywords: quasiperiodic ω-words, subword complexity, Hausdorff measure

In his tutorial [8], Solomon Marcus discussed some open questions on quasiperiodic infinite words. Soon after its publication, Levé and Richomme [7] gave answers on some of the open problems. In connection with Marcus' Question 2, they presented a quasiperiodic infinite word (with quasiperiod aba) of exponential subword complexity, and they posed the new question of what is the maximal complexity of a quasiperiodic infinite word.

In a recent paper [11], we estimated the maximal asymptotic (in the sense of [17]) subword complexity of quasiperiodic infinite words. More precisely, it is shown in [11] that every quasiperiodic infinite word ξ has at most

$$f(\xi, n) \leq O(1) \cdot t_P^n$$

factors (subwords) of length n, where t_P is the smallest Pisot number, that is, the unique positive root of the polynomial $t^3 - t - 1$. Moreover, the general construction