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PARETO GRAMMARS

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Dedicated to Prof. Gerhard Schwödiauer on the Occasion of his 80th Birthday

ABSTRACT

We study grammatical counterparts of concepts of welfare economics as Pareto optimum and Pareto sets. As targets, we consider the numbers of nonterminals and productions of a grammar.

For a context-free grammar $G = (N, T, P, S)$, we set $p(G) = (\mathrm{card}(N), \mathrm{card}(P))$. We say that the pair (m, n) is a Pareto optimum for L if there is a context-free grammar G with $L(G) = L$ and $p(G) = (m, n)$ and there is no context-free grammar G' with $L(G') = L$, $p(G') = (m', n')$, and either $m' < m$ and $n' \leq n$ or $n' < n$ and $m' \leq m$. The Pareto set $P(L)$ of L is the set of all Pareto optima of L.

We give some properties of Pareto optimal pairs and present a method to determine the Pareto set. Furthermore, we show that, for each natural number m, there is a language whose Pareto set has exactly m elements.

Keywords: context-free grammars, Pareto optimum, Pareto set

1. Introduction and Definitions

In welfare economics, the concept of a Pareto optimum (sometimes also called Pareto efficiency) is a central concept, which was introduced by Vilfredo Pareto (1848 – 1923) in his handbook of political economy ([9]). A state is Pareto optimal if there is no reallocation that makes the target property of one individual better off without making the property of another worse off.

More formally: Let $A_1, A_2, \ldots, A_n$ be sets. Moreover, for $1 \leq i \leq n$, let $\succeq_i$ be a reflexive total order on A_i. We set $A = A_1 \times A_2 \times \cdots \times A_n$. A tuple $x = (x_1, x_2, \ldots, x_n) \in A$ is called a Pareto optimum in A if there is no tuple $y = (y_1, y_2, \ldots, y_n) \in A$ such that $y_i \succeq_i x_i$ for $1 \leq i \leq n$ and there is a j, $1 \leq j \leq n$, with $y_j \succ_j x_j$. [1]

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[1] $y \succ x$ holds if and only if $y \succeq x$ and $y \neq x$.