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RECOGNITION AND COMPLEXITY RESULTS FOR PROJECTION LANGUAGES OF TWO-DIMENSIONAL AUTOMATA

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ABSTRACT

The row projection (resp., column projection) of a given two-dimensional language L is the one-dimensional language consisting of first rows (resp., first columns) of all two-dimensional words in L. The operation of row projection has previously been studied under the name "frontier language", and previous work in this area has focused primarily on one- and two-dimensional language classes.

In this paper, we study projections of languages recognized by various twodimensional automaton classes. We show that both the row and column projections of languages recognized by (four-way) two-dimensional automata are exactly contextsensitive, and we obtain a similar characterization for row and column projections of two-dimensional local languages. We also show that the column projections of languages recognized by unary three-way two-dimensional automata can be recognized using nondeterministic logspace. Finally, we study the state complexity of projection languages for two-way two-dimensional automata, focusing on the language operations of union and diagonal concatenation.

 $K\!eywords:$ language classes, projection languages, space complexity, three-way automata, two-dimensional automata, two-way automata

1. Introduction

A two-dimensional word, also known in the literature as a picture, is a generalization of the notion of a word from a one-dimensional string to a two-dimensional array or matrix of symbols. Two-dimensional words are used as the input to two-dimensional

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