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WADGE DEGREES OF CLASSES OF ω-REGULAR k-PARTITIONS

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ABSTRACT

We develop a theory of k-partitions of the set of infinite words recognizable by classes of finite automata. The theory enables to complete proofs of existing results about topological classifications of the (aperiodic) ω-regular k-partitions, and provides tools for dealing with other similar questions. In particular, we characterise the structure of Wadge degrees of (aperiodic) ω-regular k-partitions, prove the decidability of many related problems, and discuss their complexity.

Keywords: Wadge reducibility, regular k-partition, acceptor, transducer, determinacy, iterated labeled tree, fine hierarchy

1. Introduction

Working in descriptive set theory, W. Wadge [41] has shown that the degree structure of Borel sets of ω-words over any finite non-unary alphabet under the many-one reducibility by continuous functions is semi-well-ordered (i. e., it is well founded and has no 3 pairwise incomparable elements). Working in automata theory independently of W. Wadge, K. Wagner [42] has shown that the structure of regular ω-languages under the continuous reducibility is semi-well-ordered with the corresponding ordinal $\omega^\omega = sup\{\omega, \omega^2, \omega^3, \ldots\}$. Working in computability theory independently of W. Wadge and K. Wagner, the author [22] discovered a semi-well-ordered structure of "natural" m-degrees with the corresponding ordinal $\varepsilon_0 = sup\{\omega, \omega^\omega, \omega^{\omega^\omega}, \ldots\}$. In [23] (see also [24, 28]), we characterised the initial segments of the structure in [22]

This paper completes the conference papers [31, 34] by providing full details for technically involved proofs that were only sketched, and by developing a general approach to other similar problems.

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