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STATE COMPLEXITY OF GF(2)-INVERSE AND GF(2)-STAR ON BINARY LANGUAGES

Alexander Okhotin [(A)] Elizaveta Sazhneva [(A,B)]

[(A)] *Department of Mathematics and Computer Science, St. Petersburg State University*
7/9 Universitetskaya nab., Saint Petersburg 199034, Russia
alexander.okhotin@spbu.ru sazhneva.eliza@yandex.ru

[(B)] *Department of Mechanics and Mathematics, Moscow State University*
1 Leninskie gory, Moscow 119991, Russia

ABSTRACT

The GF(2)-inverse operation on formal languages, defined for every language containing the empty string, is known to preserve regularity; its state complexity is $2^n + 1$ for alphabets with at least three symbols, and $2^{n-1} + 1$ for a one-symbol alphabet. In this paper, it is proved that, for a two-symbol alphabet, its state complexity is exactly $\frac{3}{4}2^n+3$. For a more general operation, the GF(2)-star, which is applicable to every language, it is proved that its state complexity for a binary alphabet remains $2^n + 1$.

Keywords: GF(2)-concatenation, GF(2)-inverse, state complexity, primitive polynomials

1. Introduction

GF(2)-operations on formal languages were recently defined by Bakinova et al. [1]. These operations are variants of the classical concatenation and Kleene star, in which the disjunction in the definition is replaced with exclusive OR. Consider that the classical concatenation of languages K and L is the set of all such strings w, that there exists *at least one* partition $w = uv$ with $u \in K$ and $v \in L$; this is a disjunction of $|w| + 1$ conjunctions. Replacing this disjunction with exclusive OR leads to the following new operation called *GF(2)-concatenation*:

$$K \odot L = \{\ w \mid \text{the number of partitions } w = uv, \text{ with } u \in K \text{ and } v \in L, \text{ is odd }\}$$

Similarly, the Kleene star L^* is defined as the set of all strings, for which there is *at least one* partition into a concatenation of substrings from L; a similar modification leads to another new operation, the *GF(2)-star*:

$$L^{\circledast} = \{\ w \mid \text{the number of partitions } w = u_1 \dots u_k, \text{ with } k \geqslant 0 \text{ and } u_1, \dots, u_k \in L \setminus \{\varepsilon\}, \text{ is odd }\}$$

Alexander Okhotin: 0000-0002-1615-2725