Journal of Automata, Languages and Combinatorics **28** (2023) 1–3, 27 – 57

STATE COMPLEXITY BOUNDS FOR THE COMMUTATIVE CLOSURE OF GROUP LANGUAGES

Stefan Hoffmann

Informatikwissenschaften, FB IV, Universität Trier
Universitätsring 15, 54296 Trier, Germany
hoffmanns.tcs@gmail.com

ABSTRACT

We construct an automaton for the commutative closure of a given group language. The number of states of the resulting automaton is bounded by the number of states of the original automaton, raised to the power of the alphabet size, times the product of the order of the letters, viewed as permutations of the state set. This gives the asymptotic state bound $\exp(|\Sigma|(1+o(1))\sqrt{n\ln(n)})$, if the original regular language is accepted by an automaton with n states. Depending on the automaton in question, we label points of $\mathbb{N}_0^{|\Sigma|}$ by subsets of states and introduce unary automata which decompose the thus labelled grid. Based on these constructions, we give a general regularity condition, which is fulfilled for group languages.

Keywords: state complexity, commutative closure, group language, permutation automaton

1. Introduction

The area of state complexity asks for sharp bounds on the size of resulting automata for regularity-preserving operations. This question goes back at least to work by Maslov [34], but, starting with the work [42], has revived at the end of the last millennium. The class of deterministic and complete automata is the most natural, or prototypical, class. But state complexity questions have also been explored for non-deterministic automata, or other automata models, see for example the surveys [15, 25, 26]. For a more recent work that tries to unify certain constructions giving sharp state complexity bounds for deterministic and complete automata see [5].

As the number of states of a recognizing automaton could be interpreted as the memory required to describe the accepted language and is related to the runtime of algorithms employing regular languages, obtaining state complexity bounds is a natural question with applications in verification, natural language processing or software

0000-0002-7866-075X