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r-STIRLING NUMBERS OF THE SECOND KIND THROUGH CONTEXT-FREE GRAMMARS

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ABSTRACT

A context-free grammar G over an alphabet Σ is defined as a set of substitution rules that replace a letter in Σ by a formal function over Σ. In this paper, we introduce a connection between the context-free grammar $G = \{a \to ab\ ; b \to b\}$ and r-Stirling numbers of the second kind. Some combinatorial identities involving r-Stirling numbers of the second kind and binomial coefficients will be obtained by grammatical methods.

Keywords: r-Stirling numbers of the second kind, formal derivative operator, context-free grammars

1. Introduction

Let Σ be an alphabet, whose letters are regarded as independent commutative indeterminates. Following [3], a formal function over Σ is defined recursively as follows:

(i) Every letter in Σ is a formal function.

(ii) If u, v are formal functions, then $u + v$ and uv are formal functions.

(iii) If $f(x)$ is an analytic function, and u is a formal function, then $f(u)$ is a formal function.

(iv) Every formal function is constructed as above in a finite number of steps.

A context-free grammar G over Σ is defined as a set of substitution rules, called productions, replacing a letter in Σ by a formal function over Σ. For each $a \in \Sigma$, a grammar G contains at most one production of the form $a \to w$. There is here no distinction between terminals and non-terminals, as it is usual in the theory of formal languages.

Definition 1. Given a context-free grammar G over Σ, the formal derivative operator D, with respect to G, is defined in the following way:

(i) For u, v formal functions,

$$D(u + v) = D(u) + D(v) \quad \text{and} \quad D(uv) = D(u)v + uD(v).$$