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COUNTING BORDER EDGES IN BARGRAPHS

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ABSTRACT

In this paper, we study the number of bargraphs with n cells and m columns according to the statistics E_1, E_2, E_3, E_4, where E_i counts number of cells having exactly i edges in the border. Furthermore, we find the generating function for the total perimeter over all the bargraphs of n, as well as its asymptotic behavior.

Keywords: bargraph, generating function, statistic, border edge, perimeter

1. Introduction

A bargraph is a lattice path in $\mathbb{N}_0^2$, that starts at the origin and ends upon the first return to the x-axis. Each step is either an up step $(0, 1)$, a horizontal step $(1, 0)$, or a down step $(0, -1)$. The first step has to be an up step and the horizontal steps must all lie above the x-axis. An up step cannot follow a down step and vice versa. Clearly, the number of down steps must equal the number of up steps.

The appearance of bargraphs can be traced back to [15], where the authors, presented a solution of a linear solid-on-solid (SOS) model, a problem belonging to the field of statistical physics. They observed that the problem of SOS walks that do not touch the surface other than last time is equivalent to the problem of enumerating bargraph polygons according to perimeter and area. The 'perimeter' statistic, was the motivation for further research, such as paper [7], where it is studied the generating function of site perimeter (defined to be the number of nearest-neighbour vacant cells) of bargraphs as well as its asymptotic behaviour. This statistic partially inspired the present paper. Other results in relation to bargraphs appear in [2, 3, 4, 5, 6, 11, 13], where the authors study several statistics on bargraphs, such as peaks, levels, walls, etc and so on. We refer the reader to [12] to find a survey on bargraphs.

Recently, in [8], authors found a (trivial) bijection between bargraphs and Motzkin paths without peaks or valleys. Using the recursive structure of Motzkin paths they enumerated bargraphs with respect to several statistics (height of the first column,