Journal of Automata, Languages and Combinatorics **27** (2022) 4, 271–307 (© Institut für Informatik · Justus-Liebig-Universität Giessen

WEIGHTED TREE GENERATING REGULAR SYSTEMS OVER STRONG BIMONOIDS WITH REDUCTION SEMANTICS

Dávid Kószó

Department of Foundations of Computer Science, University of Szeged Árpád tér 2, Szeged, 6720, Hungary koszod@inf.u-szeged.hu

ABSTRACT

We advocate a new semantics, called reduction semantics, for tree generating regular systems and show that it is equivalent to the original semantics. We introduce the concept of weighted tree generating regular systems over strong bimonoids with reduction semantics. We show that (1) weighted tree generating regular systems over the Boolean semiring are equivalent to tree generating regular systems, and (2) weighted tree generating regular systems over semirings are equivalent to weighted tree automata over semirings.

 $K\!eywords:$ strong bimonoid, semiring, weighted tree automaton, tree generating regular system, weighted tree generating regular system

1. Introduction

Classical finite-state tree automata (for short: fta) [37, 39, 40, 41] were invented to recognize sets of trees over some ranked alphabet. A set of trees recognized by an fta is called a recognizable tree language. These fta are appropriate devices to describe qualitative properties of recognizable tree languages, such as emptiness, finiteness, membership, inclusion, and equivalence. For surveys on the theory of fta, we refer to [21, 29, 8].

In parallel and later, further concepts were introduced and proved to be equivalent to fta such as tree generating regular systems (for short: tgrs) [6]; rational tree languages [42, 21, 29]; monadic second-order logic for trees [42, 9]; regular tree grammars [6]; representable tree languages [29]. Moreover, each tree language recognized by an fta is the image of a local tree language under a deterministic tree relabeling [40, 21, 29].

Later the idea came up to describe not only qualitative but also quantitative properties of recognizable tree languages, like degree of ambiguity or costs of acceptance. Each quantitative property is a mapping from the set of input trees to the carrier