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A CHARACTERIZATION OF TOTALLY COMPATIBLE AUTOMATA

David Fernando Casas Torres

Institute of Natural Sciences and Mathematics, Ural Federal University
Pr. Lenina 51, 620000 Ekaterinburg, Russia
dkasastorres@urfu.ru

ABSTRACT

Every function on a finite set defines an equivalence relation and, therefore, a partition called the kernel of the function. Automata such that every possible partition is the kernel of a word are called totally compatible. A characterization of such automata is given together with an algorithm to recognize them in polynomial running time with respect to the number of states.

Keywords: automata, patition set

1. Background and Motivation

A *Deterministic Finite Automaton* (DFA), here simply called an automaton, is a triple $\mathcal{A} = (Q, \Sigma, \delta)$. Both Q and Σ are finite sets and $\delta\colon Q \times \Sigma \to Q$ is a function that describes the action of the *alphabet* Σ on the *state* set Q. Fixing a letter $a \in \Sigma$ in the second argument of δ produces a transformation $\delta_a\colon Q \to Q$ that is defined as $\delta_a(q) := \delta(q, a)$; here this will be denoted with right notation, i. e., $\delta_a(q) = q \cdot a$. Let Σ^* be the set of all words of the alphabet Σ (including the empty word, denoted by ε). The action of a word $w \in \Sigma^*$ over Q can be defined recursively: $q \cdot \varepsilon := q$ for every state q; if $w = va$ with $v \in \Sigma^*$ and $a \in \Sigma$, then for every state $q \in Q$, it is $q \cdot w := (q \cdot v) \cdot a$. That is why, when discussing automata, we consider just their states and alphabet. Note how any word $w \in \Sigma^*$ produces a transformation over the set of states, this is, the composition of the transformations defined by each letter.

If $P \subseteq Q$ is a nonempty set of states, its image under w is the set

$$P \cdot w := \{\, p \cdot w \mid p \in P \,\}.$$

The word $w \in \Sigma^*$ *synchronizes* a set of states $P \subseteq Q$ if $|P \cdot w| = 1$, i. e., the transformation sends every state of P to a single state. An automaton for which

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