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## 1-LIMITED AUTOMATA: WITNESS LANGUAGES AND TECHNIQUES

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## ABSTRACT

1-limited automata are single-tape Turing machines with strong rewriting restrictions, that do not allow them to recognize more than regular languages. However, 1-limited automata can be significantly more succinct than equivalent finite automata: in fact, the size gap from 1-limited automata to one-way deterministic finite automata is double exponential. In this paper we present and discuss some languages which can be used as witnesses of the gaps between different kinds of 1-limited automata and finite automata. Among them, refining previous techniques, we show that a language proposed long time ago by Meyer and Fischer as a witness of the optimality of the subset construction for finite state automata, can also be used as witness of all the currently known size gaps between 1-limited automata and different variants of finite automata. We also discuss some open problems and possible further lines of investigation.

Keywords: limited automata, regular languages, descriptional complexity

## 1. Introduction and Preliminaries

The investigation of computational models working with restricted resources is a classical topic of theoretical computer science, connecting fundamental areas such as computability and complexity. In this field of research, Thomas Hibbard introduced long time ago *limited automata*, a kind of single-tape Turing machine with rewriting restrictions [2]. This model has been recently reconsidered and deeply investigated in a series of papers (e. g., [8, 9, 4, 3, 1, 7, 10, 16]). However, many problems related to it are still open and deserve further investigations.

Limited automata are nondeterministic single-tape Turing machines which can overwrite the contents of each tape cell only in the first d visits, for a fixed constant  $d \ge 0$  (we use the name *d-limited automaton* to explicitly mention the con-

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