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PRIMITIVITY OF ATOM WATSON-CRICK FIBONACCI WORDS

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ABSTRACT

"Fibonacci strings" were first defined by Knuth in his 1968 "The Art of Computer Programming," as being an infinite sequence of strings obtained from two initial letters $f_1 = a$ and $f_2 = b$, by the recursive definition $f_{n+2} = f_{n+1} \cdot f_n$, for all positive integers $n \geq 1$, where "·" denotes word concatenation. Motivated by theoretical studies of DNA computing, several generalizations of Fibonnaci words have been proposed under the umbrella term involutive Fibonacci words. These include ϕ-Fibonacci words and indexed ϕ-Fibonacci words, where ϕ denotes either a morphic or an antimorphic involution. (In the particular case of the DNA alphabet $\Delta = \{A, C, G, T\}$, where ϕ is the Watson-Crick complementarity (antimorphic) involution on Δ^* that maps A to T, G to C, and vice versa, the ϕ-Fibonacci words are termed atom Watson-Crick Fibonacci words.) In this paper, we investigate the properties of atom ϕ-Fibonacci words over a four-letter alphabet, whereby "atom" indicates that the two initial words are singleton letters. The results are different from the case of the classical Fibonacci words over a two-letter alphabet, which are all primitive, in that for some (anti)morphic involutions, some initial letters, and some indices n, we have that the n-th atom ϕ-Fibonacci word is primitive, while for some others it is not. In the particular case of the Watson-Crick complementarity antimorphic involution, regardless of the initial two letters in the Fibonacci recursion (different, or the same), for all $n > 3$, the n-th atom Watson-Crick Fibonacci word is primitive.

Keywords: DNA computing, Watson-Crick complementarity, antimorphic involution, atom Fibonacci words, involutive Fibonacci words

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