Journal of Automata, Languages and Combinatoric<br/>s ${\bf 26}$ (2021) 3–4, 197–220 © Institut für Informatik · Justus-Liebig-Universität Giessen

## OGDEN'S LEMMA FOR SUBCLASSES OF RANDOM CONTEXT GALLERIES

SIGRID EWERT JOY IDAHOSA

## School of Computer Science and Applied Mathematics, University of the Witwatersrand, Johannesburg, Private Bag 3, Wits, 2050, South Africa Sigrid.Ewert@wits.ac.za joy.idahosa1@gmail.com

## ABSTRACT

Random context picture grammars are used to generate pictures through successive refinement. There exist several subclasses of these grammars, e.g., context-free picture grammars, random permitting context picture grammars, random forbidding context picture grammars and table-driven context-free picture grammars. These classes generate context-free galleries (cfpls), random permitting context galleries (rPcpls), random forbidding context galleries (rPcpls), random forbidding context galleries (rFcpls) and table-driven context-free galleries (Tcfpls), respectively. For all these classes of galleries, necessary conditions have been proven. In particular, for cfpls, there exists a pumping-shrinking lemma, for rPcpls, a pumping lemma and for rFcpls, a shrinking lemma. For Tcfpls, two necessary conditions have been proven. Recently, a new necessary condition related to the size of a subpicture was proven for each of the abovementioned classes of galleries. We now prove theorems that are an alternative to the existing necessary conditions. This is done by adapting Ogden's idea of marking parts of a word for the picture case. We illustrate the new conditions with examples. For rPcpls and Tcfpls, we also give examples of galleries for which the marking is necessary.

 $K\!eywords:$  formal language, regulated rewriting, random context grammar, picture grammar, necessary condition, Ogden's lemma

## 1. Introduction

Syntactic methods of picture generation have become established during the last decades. A comprehensive survey of the area of picture languages in 1993 is given in [19]. Broadly speaking, existing methods use either successive addition or successive refinement to generate pictures. In the former category, recently developed models include cooperating context-free array grammar systems with permitting features [21] and several 2D picture array generating models based on membrane computing [20]. In the latter category, developments after 1993 include grid picture grammars [3] and table-driven context-free picture grammars [1].

This work is based upon research supported by the National Research Foundation (of South Africa). Any opinion, findings and conclusions or recommendations expressed in this material are those of the authors and therefore the NRF does not accept liability in regard thereto.