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CONSTRUCTING REVERSIBLE TURING MACHINES IN A REVERSIBLE AND CONSERVATIVE ELEMENTARY TRIANGULAR CELLULAR AUTOMATON

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ABSTRACT

We study the problem of how we can construct reversible Turing machines (RTMs) compactly in a two-dimensional reversible cellular automaton (CA). The CA model used here is an elementary triangular partitioned CA (ETPCA) having an extremely simple local transition function. It has been shown that any RTM is realized in a reversible and non-conservative ETPCA No. 0347, where 0347 is an identification number in the class of 256 ETPCAs. In this paper, we show that it is also possible to construct RTMs in a reversible and conservative ETPCA 0137, which has very different features from ETPCA 0347, by a systematic and hierarchical manner. Here, a reversible logic element with memory (RLEM), rather than a reversible logic gate, is used in the basic level of the construction. In particular, a special type of an RLEM No. 4-31 is implemented in ETPCA 0137. Then RTMs are constructed using the RLEM pattern. By this, the construction of configurations that simulates RTMs is greatly simplified.

Keywords: reversible computing, elementary triangular partitioned cellular automaton, reversible logic element with memory, reversible Turing machine, universality

1. Introduction

We investigate the problem of how reversible Turing machines (RTMs) are simulated in a simple two-dimensional reversible cellular automaton (CA). Here we use an *elementary triangular partitioned CA* (ETPCA) [3, 5, 8, 9] as a CA model. Its triangular cell has three parts each of which has two states. It is defined by only four local transition rules, and thus it is extremely simple. There are 256 ETPCAs in total, and they form one of the simplest subclasses of 2D CAs. In [8] three kinds of dualities on ETPCAs, such as the duality under reflection (i. e., taking the mirror image) and others, are defined, and the 256 ETPCAs are classified into 82 equivalence classes based on them.

It is known that there are 12 equivalence classes of reversible ETPCAs. Among them, three classes are computationally universal [3, 5, 8, 9]. The representatives of

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