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SOME RESULTS ABOUT MAXIMAL PRIMITIVE SETS

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ABSTRACT

A result of a student of Erdős is well known: *for every integer $n \geq 1$, each subset of $I(2n) = \{1, 2, \ldots, n+1, n+2, \ldots, 2n\}$ having size $n+1$ contains at least two distinct elements of which the smaller divides the larger.* This is proved using the pigeonhole principle. In strict relation with this result, here we study the maximal primitive sets and improve the results of a previous paper.

Keywords: n-tuple, divisor, multiple, primitive sets

1. Introduction

This article was conceived during a course (TFA 2015, University of Florence, Italy) given by G. Pirillo. C. Bindi was a student of that course and M. Pellegrini has been a PhD student of G. Pirillo in a previous year.

During the 2015 TFA course a problem posed by Erdős has been presented along with two of its solutions: the original one (see [1]) and the one developed by a student of St. Petersburg (see [5]). Furthermore, G. Pirillo and the students of the TFA course produced a third demonstration.

During a dinner Paul Erdős posed a question to one of his students, Lajos Pósa: is it true that, for every integer $n \geq 1$, each subset of $I(2n) = \{1, 2, \ldots, n+1, n+2, \ldots, 2n\}$ having size $n+1$ contains at least two distinct elements of which the smaller divides the larger? Before the dinner ended, the student proved this fact using the pigeonhole principle and equivalence classes of different cardinalities.

In strict relation with this result there is the following question: *what can we say about the subsets of $I(2n)$ having exactly cardinality n?* There is already some research and results about these subsets, see for instance [7, 3, 4, 6].