Journal of Automata, Languages and Combinatorics **25** (2020) 4, 291 – 320

A COMBINATORIAL APPROACH FOR THE STATE COMPLEXITY OF THE SHUFFLE PRODUCT

Pascal Caron, Jean-Gabriel Luque, Bruno Patrou

LITIS, Université de Rouen
76801 Saint-Étienne du Rouvray Cedex, France
{Pascal.Caron,Jean-Gabriel.Luque,Bruno.Patrou}@univ-rouen.fr

ABSTRACT

We investigate the state complexity of the shuffle operation on regular languages initiated by Câmpeanu *et al.* and studied subsequently by Brzozowski *et al.*. We shift the problem into the combinatorics domain by turning the problem of state accessibility into a problem of intersection of partitions. This allows us to develop new tools and to reformulate the conjecture of Brzozowski *et al.* about the above-mentionned state complexity.

Keywords: state complexity, automata theory, combinatorics

1. Introduction

Studies on state complexity have been going on for more than forty years now. The seminal work of Maslov [15], which gives values (without proofs) for the state complexity of some operations: square root, cyclic shift and proportional removal, paves the way. From these foundations, a very active field of research was open mainly initiated by Yu et al. [20]. Lots and lots of papers were produced and different sub-domains appeared depending on whether the used automata are deterministic or not, whether the languages are finite or infinite, belongs to some classes (codes, star-free, ...) and so on. We focus here on the (complete) deterministic case for any language.

The state complexity of a rational language is the size of its minimal (complete deterministic) automaton and the state complexity of a rational operation is the maximal one of those languages obtained by applying this operation onto languages of fixed state complexities.

The classical approach is to compute an upper bound and to provide a witness, that is a specific example reaching the bound which is then the desired state complexity.

In some cases, the classical method has to be enhanced by an algebraic approach consisting in building a witness for a certain class of rational operations by searching in a set of automata with as many transition functions as possible. This method has the advantage of being applied to a large class of operations and has been described