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COUNTING PERMUTATIONS BY THE NUMBER OF VERTICAL EDGES IN THEIR BARGRAPHS

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ABSTRACT

In this paper, we consider a new statistic on the set $\mathcal{S}_n$ of permutations of length n which records the number of vertical edges within their bargraph representations. Indeed, we determine the distribution of this statistic on a class of multi-permutations having $\mathcal{S}_n$ as a subset. We compute an explicit formula for the total number of vertical edges within all members of this class and also for the number of interior vertices in the corresponding bargraphs. A generating function formula is determined and some closely related statistics are considered. Finally, we show that the total number of vertical edges in all the members of $\mathcal{S}_n$ equals the number of runs in the members of $\mathcal{S}_{n+1}$ by constructing an explicit bijection which applies more generally to multi-permutations.

Keywords: bargraphs, multi-set permutations, generating functions, permutation statistics

1. Introduction

Recall that a *bargraph* is a self-avoiding random walk in the first quadrant starting from the origin and terminating on the x-axis and lying strictly above the x-axis except for the endpoints, where the permissible steps are the up step $u = (0, 1)$, down step $d = (0, -1)$ and horizontal step $h = (1, 0)$. Note that no u can follow a d, and vice versa, and each h must lie (strictly) above the x-axis. Bargraphs, also referred to as wall polyominoes [4] or skylines [6], have recently been studied from several

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