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RESTRICTED LEGO TOWERS

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ABSTRACT

Two particular families of polyominoes are introduced and enumerated, namely so-called *k-Lego towers* and *cone k-Lego towers*. The first family consists of Lego towers containing in each floor exactly one piece having length less than or equal to k. The generating function for the number of k-Lego towers as well as cone k-Lego towers with n floors according to a statistic given by perimeter and area is determined. Furthermore, for the number of cone k-Lego towers with exactly n floors an explicit formula is derived. In addition, several special cases are treated in detail.

Keywords: Lego towers, generating functions, perimeter, area

1. Introduction

In the plane $\mathbb{Z} \times \mathbb{Z}$, a *cell* is a unit square whose vertices have integer coordinates. A *polyomino* is a set of cells in the plane $\mathbb{Z} \times \mathbb{Z}$ connected by their edges, defined up to translations. Polyominoes are sometimes also called (fixed) *animals*, a notion firstly used in [25].

Polyominoes were introduced by Golomb, see [9]. Since then, various families of polyominoes have been defined and were investigated in several research directions. However, the problem of their enumeration in the general case remains unsolved. Even though the idea of polyominoes was introduced in the middle of the last century, polyominoes continue to be a very active research area, appearing in many branches of mathematics and physics.

Let us give some references. The enumerative aspects of polyominoes can be traced back to [7] where the authors construct a bijection between convex polyominoes and words of an algebraic language, thereby determining the number of convex polyominoes with perimeter $2n + 8$. For more recent enumerative results, we refer to [11]