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A MULTISET ANALOGUE OF COFACTORS

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ABSTRACT

In this paper, we first review the graph-theoretical interpretations of the determinant and cofactors of a matrix, using the idea of cycle covers of the associated digraph of that matrix. We then also review the multiset analogue of the combinatorial interpretation of the determinant based on the idea of Lydon covers. As the main result of this paper, we also give a multiset analogue of the cofactor of any entry of a matrix by giving a generalization of the concept of the Lydon cover. We then obtain a multiset analogue of the well-known Cayley-Hamilton theorem, as an application of the main result of this paper. Finally, we conclude the paper with several interesting open problems and conjectures.

Keywords: multiset analogue, cofactor, cycle cover, Lyndon cover, Cayley-Hamilton theorem

1. Introduction

The determinant of a matrix was originally invented to find solutions of systems of linear algebraic equations. The initial efforts were made by many great mathematicians to formulating and proving the so-called *Cramer's rule*, *Laplace expansion*, and *Cauchy-Binet determinantal identity*, and these led them to find many interesting algebraic identities [3]. The basic idea behind all these determinantal identities is to express the determinant of a matrix in terms of determinants of lower orders. These subdeterminants together with their corresponding signs are called *cofactors*.

From algorithmic viewpoint finding combinatorial interpretations of determinants are more desirable than their linear algebraic manipulations for designing efficient parallel algorithms for computing the determinant [12]. In particular, in the graph-theoretical setting, the computation of the determinant of a matrix A amounts to evaluating the signed weighted sum of cycle covers [2] of its associated digraph $D(A)$. Moreover, cofactors are also interpreted as the signed weighted sum of generalized cycle covers [11].

Lyndon words play a central role in theory of combinatorics of words [6] as a subfield of formal languages and automata theory [8]. The author in [11] has already obtained a multiset analogue of the classic determinant of a matrix by extending the