THE FIELD OF THE REALS AND THE RANDOM GRAPH ARE NOT FINITE-WORD ORDINAL-AUTOMATIC[1]

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ABSTRACT

Recently, Schlicht and Stephan lifted the notion of automatic-structures to the notion of (finite-word) ordinal-automatic structures. These are structures whose domain and relations can be represented by automata reading finite words whose shape is some fixed ordinal α. We lift Delhommé's relative-growth-technique from the automatic and tree-automatic setting to the ordinal-automatic setting. This result implies that the random graph is not ordinal-automatic and infinite integral domains are not ordinal-automatic with respect to ordinals below $\omega_1 + \omega^\omega$ where ω_1 denotes the first uncountable ordinal.

Keywords: Ordinal-automatic structures, automatic integral domains, Rado graph, growth rates

1. Introduction

Finite automata play a crucial role in many areas of computer science. In particular, finite automata have been used to represent certain infinite structures. The basic notion of this branch of research is the class of automatic structures (cf. [12, 6]). A structure is automatic if its domain as well as its relations are recognised by (synchronous multi-tape) finite automata processing finite words. This class has the remarkable property that the first-order theory of any automatic structure is decidable. One goal in the theory of automatic structures is a classification of those structures that are automatic (cf. [4, 14, 13, 11, 15]). Besides finite automata reading finite or infinite (i.e., ω-shaped) words there are also finite automata reading finite or infinite *trees*. Using such automata as representation of structures leads to the notion of tree-automatic structures [2]. The classification of tree-automatic structures is less advanced but some results have been obtained in the last years (cf. [4, 7, 9]). Schlicht and Stephan [16] and Finkel and Todorčević [5] have started research on a new branch of automatic structures based on automata processing α-words where α is some ordinal. An α-word is a map $w \in \Sigma^\alpha$ for some finite alphabet Σ. We call w a finite α-word if there is one symbol $\diamond$ such that $w(\beta) = \diamond$ for all but finitely many ordinals $\beta < \alpha$. We call the structures represented by finite-word α-automatic structures (α)-automatic. Many of the fundamental results on automatic structures have analogues in the setting of (α)-automatic structures.