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NONDETERMINISTIC STATE COMPLEXITY OF POSITIONAL ADDITION [1]

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ABSTRACT

Consider nondeterministic finite automata recognizing base-k positional notation of numbers. Assume that numbers are read starting from their least significant digits. It is proved that if two sets of numbers S and T are represented by nondeterministic automata of m and n states, respectively, then their sum $\{s + t \mid s \in S,\ t \in T\}$ is represented by a nondeterministic automaton with $2mn+2m+2n+1$ states. Moreover, this number of states is in the worst case necessary for all $k \geqslant 6$.

Keywords: Finite automata, state complexity, descriptional complexity, addition, positional notation

1. Introduction

Descriptional complexity of operations on regular languages with respect to their representation by finite automata and regular expressions is among the common topics of automata theory. With respect to deterministic finite automata (DFAs), and using the number of states as a complexity measure, the *state complexity* of basic operations on languages was determined by MASLOV [13] in 1970. In particular, such results as "if languages K and L are recognized by DFAs of m and n states, respectively, then the language KL requires a DFA with up to $(2m-1)2^{n-1}$ states" originate from that paper.

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