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HOMOMORPHISMS PRESERVING LINEAR CONJUNCTIVE LANGUAGES

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ABSTRACT

It is proved that a homomorphism h preserves the class of languages generated by linear conjunctive grammars (equivalently, recognized by trellis automata, also known as one-way real-time cellular automata) if and only if either h is injective, or h maps every symbol to the empty string. The transformation is effective in the former case and cannot be effectively done in the latter case.

Keywords: Conjunctive grammars, trellis automata, cellular automata, closure properties, codes

1. Introduction

The family of languages studied in this paper has several equivalent definitions. It was first encountered by SMITH [12] and DYER [4] in the study of cellular automata, as the family recognized by their simplest kind, the *one-way real-time cellular automata*. A similar definition was given by ČULÍK, GRUSKA and SALOMAA [2, 3], motivated by modelling a massively parallel system with simple identical processors connected in a uniform trellis, whence the name *trellis automata*. The isomorphism of these two definitions was noticed by CHOFFRUT and ČULÍK [1]. Next, IBARRA and KIM [5] characterized this family by a specially restricted type of Turing machines. The equivalence of trellis automata to two families of formal grammars, *linear conjunctive grammars* [6, 8] and *linear Boolean grammars* [10], was established by the author. These quite different but equally natural representations place this language family among the most important ones in formal language theory.

The nonclosure of the languages recognized by trellis automata under homomorphisms, including the particular case of letter-to-letter homomorphisms, was established by ČULÍK et al. [3]. Another related result is the closure of this family under length-multiplying injective homomorphisms, independently proved by ČULÍK et al. [3] using trellis automata and by IBARRA and KIM [5] using a sequential machine characterization.

[1] This work was done during the author's studies at the School of Computing, Queen's University, Kingston, Ontario, Canada.