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ON COMPUTABILITY AND SOME DECISION PROBLEMS OF PARAMETRIC WEIGHTED FINITE AUTOMATA [1]

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ABSTRACT

Parametric weighted finite automata (PWFA) are a multidimensional variant of weighted finite automata (WFA). In contrast to WFA, one primarily considers the set of vectors computed by a PWFA and not the function it computes. In this paper we show that the membership, emptiness and equivalence problems for PWFA are recursively undecidable and that there is no algorithm that effectively minimizes a PWFA. We study the correlation between iterated function systems (IFS) and the class cPWFA of PWFA that are based on contraction mappings, where we show that it is decidable, if a PWFA belongs to cPWFA, that automata in cPWFA compute affine transformations of the attractors of certain IFS, that the set of sets computable by automata in cPWFA is effectively closed under arbitrary affine transformation and that any PWFA computing a non compact set cannot be a member of cPWFA. For PWFA with unary alphabet we proof that the set of sets computable is effectively closed under set union and that the membership problem for unary alphabet integer PWFA is decidable.

Keywords: Parametric Weighted Finite Automata, Computability, Decision Problems, Fractals

1. Introduction

Weighted finite automata (WFA, [22]) are extensions of finite automata that compute functions $f : \Sigma^* \mapsto \mathcal{S}$, where Σ is a finite alphabet and $\mathcal{S}$ is a semiring. WFA over $\mathcal{S} = \mathbb{R}$ can be adapted to compute real functions by using input words of infinite length which are interpreted as the argument of the function [7]. The computational power of WFA over $\mathbb{R}$ was examined in [8] and [9], where one of the main results is that the only smooth functions (that means every derivative is continuous) computable are polynomials. They can also display some non smooth functions, including all finite impulse response dyadic scaling functions and wavelets deducted from a multiresolution analysis [6, 26].

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