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DETERMINISTIC MOLES CANNOT SOLVE LIVENESS [1]

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ABSTRACT

We examine the conjecture that no polynomial can upper bound the increase in the number of states when a one-way nondeterministic finite automaton (1NFA) is converted into an equivalent two-way deterministic finite automaton (2DFA). We study the problem of *liveness*, which admits 1NFAs of polynomial size and is known to defy 2DFAs of polynomial size if and only if the conjecture is true. We focus on *moles*, a restricted class of two-way nondeterministic automata that includes the 1NFAs solving liveness. We show that, in contrast, 2DFA moles cannot solve liveness, irrespective of their size.

Keywords: One-way nondeterministic finite automata, two-way deterministic finite automata, Sakoda-Sipser conjecture, 2D versus 2N, descriptional complexity

1. Introduction

It has been known for a long time [17] that the power of one-way deterministic finite automata (1DFAs) does not increase when they are enhanced with nondeterminism and/or bidirectionality: be they one-way nondeterministic (1NFAs), two-way deterministic (2DFAs), or even two-way nondeterministic (2NFAs), finite automata still fail against non-regular problems. However, this describes the situation only from the point of view of computability.

From the complexity perspective, the extra capabilities do increase the power of 1DFAs, in the sense that against the same problems the enhanced automata occasionally manage to stay exponentially smaller [1, 14, 16]. This observation has initiated a more general and systematic investigation: *when we convert a machine of a particular type into an equivalent machine of a different type, how much 'larger' need the new machine be, in general?* Even the apparently simple world of regular languages hosts some most intriguing instances of this question.

The four types of automata mentioned above define a dozen different conversions (Fig. 1). The famous one is that from 1NFAs to 1DFAs. We know that every n-state

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