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STATE HIERARCHY FOR ONE-WAY FINITE AUTOMATA [1] [2]

Viliam Geffert
Department of Computer Science, P. J. Šafárik University
Jesenná 5, 040 01 Košice, Slovakia
e-mail: viliam.geffert@upjs.sk

ABSTRACT

Quite recently, it has been shown that, for each n, and each d between n and 2^n, there exists a regular language for which each optimal nondeterministic one-way finite state automaton (nfa) uses exactly n states, but its optimal deterministic counterpart (dfa) exactly d states. This gives the complete state hierarchy for the relation between nfa's and dfa's. However, in literature, either the size of the input alphabet for these automata is very large, namely, $2^{n-1}+1$, or the argument is "non-constructive," proving the mere existence without an explicit exhibition of the witness language.

We shall give a simpler "constructive" proof for this state hierarchy, displaying explicitly the witness automata and, at the same time, reduce the input alphabet size. That is, we shall present a construction of an optimal nfa with n states, and with the input alphabet size bounded by $n+2$, for which the equivalent optimal dfa uses exactly d states, for each given n and d satisfying $n \leq d \leq 2^n$.

Keywords: Descriptional complexity, finite-state automata, regular languages

1. Introduction and Preliminaries

In this paper, we shall present a more detailed analysis concerning the relation between the number of states in nondeterministic one-way finite state automata (nfa's) and their deterministic counterparts (dfa's). At first glance, this problem has been resolved completely for the one-way automata, though not all relations are known for the two-way machines.

Every student knows the classical subset construction [8], one of the oldest results, replacing a given nfa with n states by an equivalent dfa with 2^n states. Students are also told that "this exponential blow-up cannot be improved". The most popular witness example is the set consisting of all binary words with zero as the $(n-1)$-st symbol from the end. The optimal nondeterministic automaton for this language

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