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SHAPE PRESERVING BOTTOM-UP TREE TRANSDUCERS [1]

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ABSTRACT

It is easy to observe that both top-down and bottom-up (finite state) relabelling tree transducers preserve the shape of the trees. It is known that every shape preserving top-down tree transducer is semantically equivalent to a top-down relabelling tree transducer. In this way, for top-down tree transducers, the semantic property "being shape preserving" is characterized by the syntactic property "being relabelling". In this paper we prove the analogous result for bottom-up tree transducers. Namely, we show that every shape preserving bottom-up tree transducer is equivalent to a bottom-up relabelling tree transducer. We also prove that it is decidable if two shape preserving bottom-up tree transducers are equivalent.

Keywords: Tree automata, bottom-up tree transducers; top-down tree transducers

1. Introduction

A finite state transducer M (fst, cf. [2, 3]) is a system which reads a string x over the input alphabet Σ and computes a finite set $\tau_M(x)$ of strings over the output alphabet Δ. The transformation computed by M, in other words the semantics of M, is the set of pairs (x, y) such that $x \in \Sigma^*$ and $y \in \tau_M(x)$. In general, the input string x and an output string $y \in \tau_M(x)$ may have different lengths. However, if, for every input x and output $y \in \tau_M(x)$, the length of x and the length of y are equal, then M is called length preserving. For example a Mealy automaton, i. e. a fst which, scanning an input symbol $a \in \Sigma$, writes out exactly one output symbol $b \in \Delta$, is a length preserving fst. It was shown in [8] that a fst is length preserving if and only if it is semantically equivalent to a Mealy automaton.

Recently in [5] this result was generalized from strings to trees and from fst's to top-down tree transducers [4, 10]. In fact, a top-down tree transducer M is also a finite state machine. However, M reads a tree s over the input ranked alphabet Σ and computes a finite set $\tau_M(s)$ of trees over the output ranked alphabet Δ. The tree transformation computed by M, i. e. the semantics of M, is the set of pairs (s, t)

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