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REGULAR EXPRESSIONS: NEW RESULTS AND OPEN PROBLEMS^{1,2}

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ABSTRACT

Regular expressions have been studied for nearly 50 years, yet many intriguing problems about their descriptive capabilities remain open. In this paper we sketch some new results and discuss what remains to be solved.

Keywords: Regular expression, finite automaton

1. Introduction

The class of regular languages is one of the most important and best-understood classes of languages in computer science. A regular language can be represented in several different ways (without trying to be exhaustive):

- as the language accepted by
 - a deterministic finite automaton (DFA);
 - a nondeterministic finite automaton (NFA); or
 - a nondeterministic finite automaton with ϵ -transitions (NFA- ϵ);
- as the language specified by
 - a regular expression (RE), allowing the operations of union (+), concatenation (typically represented implicitly by juxtaposition), and Kleene closure (*) [29, 41, 10];
 - a generalized regular expression (GRE), allowing the additional operations of intersection (\cap) and complement (\neg) [41].

 $^{^1\}mathrm{By}$ a fatal mistake by the journal in Issue 2/3 of Vol. 9 (2004) of JALC at pages 233–256 a wrong version of the article has been published, the presented article is the correct one.

²Full version of an invited lecture presented at the 4th Workshop on *Descriptional Complexity of Automata, Grammars and Related Structures* (London, Ontario, Canada, August 21–24, 2002).