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NP PREDICATES COMPUTABLE IN THE WEAKEST LEVEL OF THE GRZEGORCZYCK HIERARCHY [1]

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ABSTRACT

Let $(\mathcal{E}_r)_{r\in\mathbf{N}}$ be the hierarchy of Grzegorczyk. Its weakest level, $\mathcal{E}_0$ is indeed quite weak, as it doesn't even contains functions such as $max(x, y)$ or $x + y$. In this paper we show that $SAT \in \mathcal{E}_0$ by developing a technique which can be used to show the same result holds for other NP problems. Using this technique, we are able to show that also the Hamiltonian Cycle Problem is solvable in $\mathcal{E}_0$.

Keywords: Grzegorczyk hierarchy, subrecursion classes, SAT, NP

1. Introduction and Notation

Why should a computer scientist be interested in the Grzegorczyk Hierarchy today? Because there are still open problems related to the lower levels of this hierarchy. Even more, solving some of these problems may lead to solutions to other open problems in complexity theory. For example, if one can prove that the inclusion $\mathcal{E}_{0*} \subseteq \mathcal{E}_{2*}$ is strict, then the Linear Time Hierarchy LTH is properly contained in $LINSPACE$, as $LTH \subseteq \mathcal{E}_{0*}$ and $LINSPACE = \mathcal{E}_{2*}$.

We shall follow the notation in Rose [8] and denote by $\mathbf{q}(x, y)$ the arithmetical quotient of the integer division of x by y, by $\mathbf{r}(x, y)$ the remainder of the integer division of x by y, by $\mathbf{d}(x)$ the *length* of the binary representation of x and by $\mathbf{D}(x, y) = 2^{\mathbf{d}(x)*\mathbf{d}(y)}$ the *smash* function.

Let $(\mathcal{E}_r)_{r\in\mathbf{N}}$ be the Grzegorczyk Hierarchy, and let P_f be the set of the polynomial-time computable functions. We denote by $\mathcal{C}_*$ the subset of the Boolean-valued functions in the function set $\mathcal{C}$. Also $(P_f)_* = P$.

Note that P_f contains the *smash* function $\mathbf{D}$, which is not in $\mathcal{E}_2$, as it is not polynomially bounded. Therefore $P_f \not\subseteq \mathcal{E}_2$.

It was already known that $\mathcal{E}_2 \not\subseteq P_f$, provided $P \neq NP$ ([2, Theorem 1]).

We shall prove a much stronger assertion, namely

$\mathcal{E}_0 \not\subseteq P_f$, provided $P \neq NP$.

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