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STATE COMPLEXITY OF SHUFFLE ON TRAJECTORIES [1]

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ABSTRACT

It is easy to get an upper bound for the state complexity of shuffle on trajectories that generalizes the bound for unrestricted shuffle. We establish improved bounds for slender trajectories, that is, trajectories which have only a constant number of strings of a given length. For trajectories with USL-index 1 (or 1-thin trajectories, those with only one string of each length) we obtain an asymptotically optimal lower bound when the state complexity of the trajectory grows with respect to the state complexity of the component languages. Some estimations are improved by considering nondeterministic state complexity.

Keywords: Descriptional complexity, finite automata, shuffle on trajectories, slender languages

1. Introduction

The notion of shuffle on trajectories was introduced as an extension of the existing notions of shuffle by Mateescu *et al.* [6] to provide an abstraction of parallel composition of words, an important operation in parallel computation.

The paper of Mateescu *et al.*, as well as the paper of Harju *et al.* [2] both give proofs that, given a regular trajectory T, and regular languages L_1, L_2, the operation $L_1 \sqcup\!\sqcup_T L_2$ always yields a regular language (for definitions, see section 2). For other research on shuffle on trajectories see, e. g. Mateescu and Salomaa [7] or Mateescu *et al.* [8].

In this paper, we consider the efficiency of such shuffle on trajectory operations, through the usual measure of *state complexity* [15]. The family of shuffle on trajectory operations is very complex, and in this paper, we only begin to address the many questions which arise from studying the state complexities of these operations. However, we incorporate other measures of complexity used in formal languages and automata theory, including nondeterministic state complexity and language density.

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